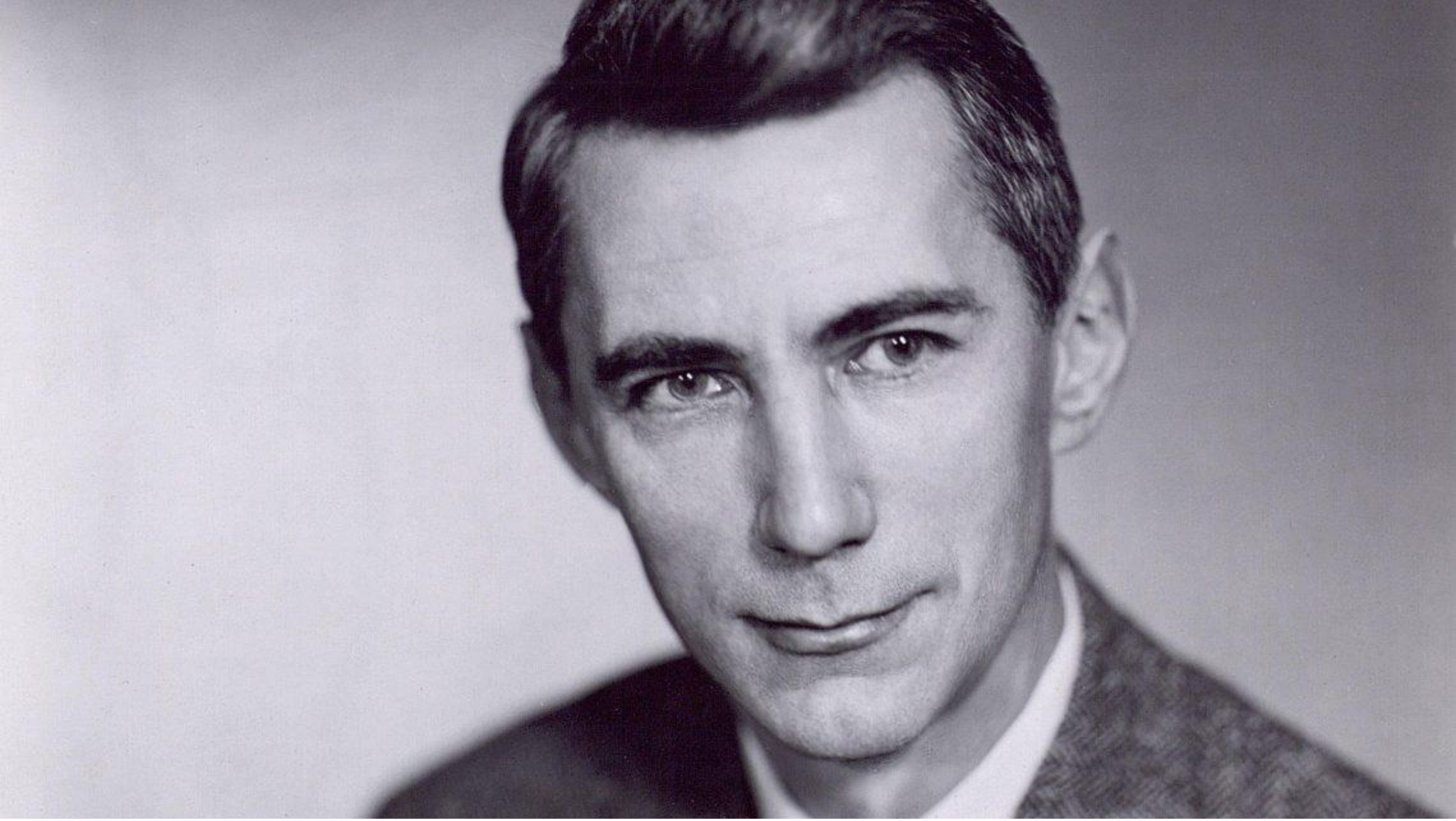
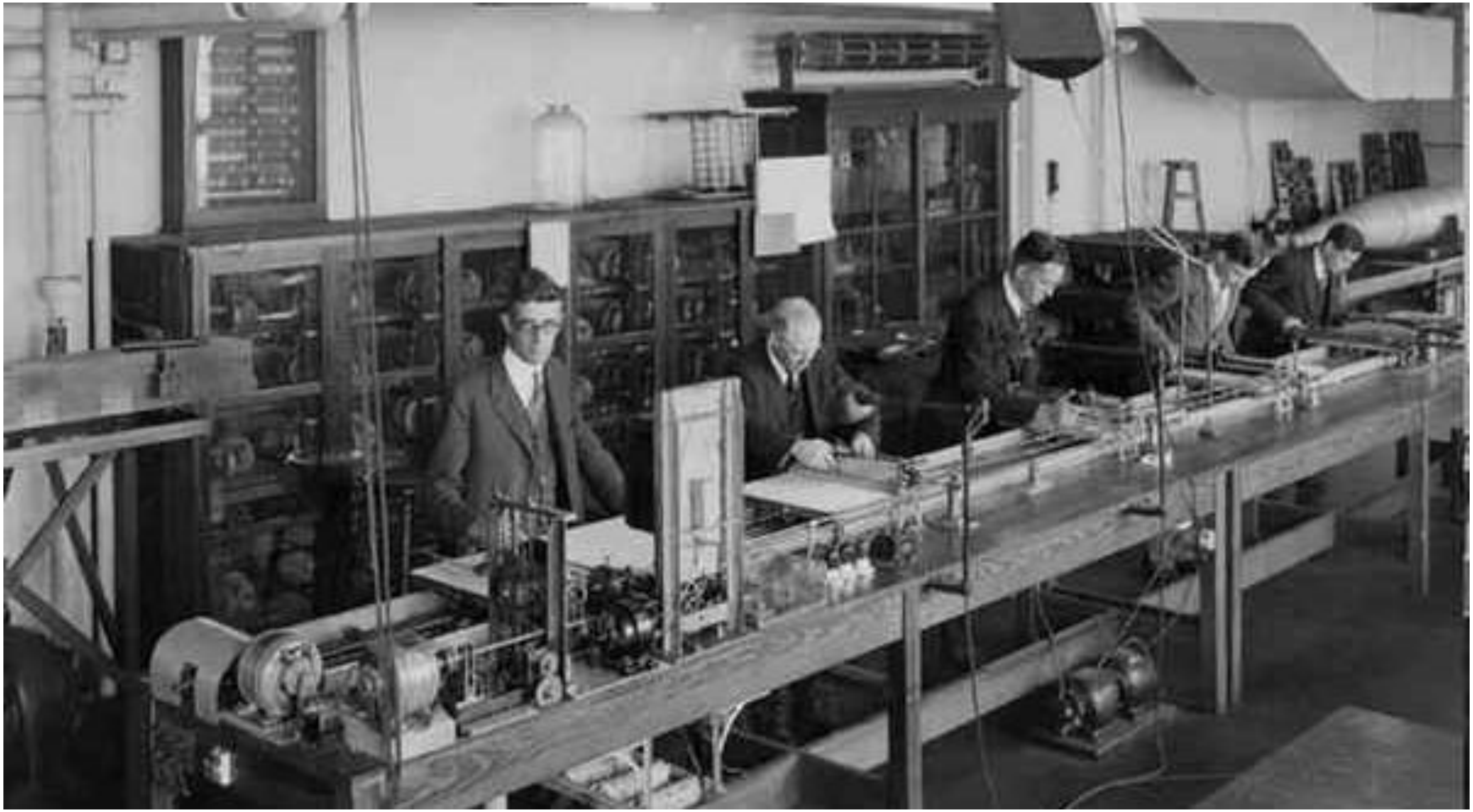


Shannon and Digital Circuits

Shannon at MIT

- In 1936, Shannon was assigned to work on the Differential Analyzer, an analog computer, controlled by relays and switches.
- Shannon was intrigued by how closely the relays' operation resembled the workings of symbolic logic.
- Shannon realized that switches, combined in circuits, carry out operations of symbolic logic, an analogy apparently never recognized before.

The Differential Analyzer



In 1937, Claude Shannon wrote his MsC thesis

A Symbolic Analysis of Relay and Switching Circuits

Harvard professor and writer, Howard Gardner called it "*possibly the most important, and also the most noted, master's thesis of the century*".

ACKNOWLEDGMENT

The author is indebted to Professor F. L. Hitchcock, who supervised the thesis, for helpful criticism and advice.

A SYMBOLIC ANALYSIS
OF
RELAY AND SWITCHING CIRCUITS

by

Claude Elwood Shannon
B.S., University of Michigan
1936

Submitted in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
from the
Massachusetts Institute of Technology

Fundamental Definitions and Postulates. We shall limit our treatment to circuits containing only relay contacts and switches, and therefore at any given time the circuit between any two terminals must be either open (infinite impedance) or closed (zero impedance).

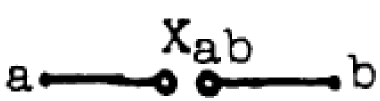


Fig. 1

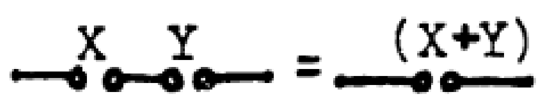


Fig. 2

Fig. 3

Postulates

1. a. $0 \cdot 0 = 0$ — A closed circuit in parallel with a closed circuit is a closed circuit.

   b. $1 + 1 = 1$ — An open circuit in series with an open circuit is an open circuit.

2. a. $1 + 0 = 0 + 1 = 1$ — An open circuit in series with a closed circuit in either order is an open circuit.

   b. $0 \cdot 1 = 1 \cdot 0 = 0$ — A closed circuit in parallel with an open circuit in either order is a closed

3. a. $0 + 0 = 0$ — A closed circuit in series with a closed circuit is a closed circuit.

   b. $1 \cdot 1 = 1$ — An open circuit in parallel with an open circuit is an open circuit.

4. At any given time either $X = 0$ or $X = 1$.

Shannon Decomposition

10. a. $f(X_1, X_2, \ldots X_n) = X_1 f(1, X_2 \ldots X_n) + X_1' f(0, X_2 \ldots X_n)$

b. $f(X_1 \ldots X_n) = [f(0, X_2 \ldots X_n) + X_1] \cdot [f(1, X_2 \ldots X_n) + X_1']$

11. a. $f(X_1 \ldots X_n) = X_1 X_2 f(1,1,X_3 \ldots X_n) + X_1 X_2' f(1,0,X_3 \ldots X_n)$

$+ X_1' X_2 f(0,1,X_3 \ldots X_n) + X_1' X_2' f(0,0,X_3 \ldots X_n)$

12. a. $f(X \ldots X_n) = f(1,1,1 \ldots 1) X_1 X_2 \ldots X_n + f(0,1,$

$1 \ldots 1) X_1' X_2 \ldots X_n + \ldots\ldots\ldots + f(0,0,0 \ldots 0)$

$X_1' X_2' \ldots X_n'$

A system for logic simplification

- Shannon devised a symbol manipulation system for the optimization of Boolean functions (and relay circuits)

14. a. $X = X + X = X + X + X = \text{etc.}$

    b. $X = X \cdot X = X \cdot X \cdot X = \text{etc.}$

15. a. $X + XY = X$

    b. $X(X + Y) = X$

16. a. $XY + X'Z = XY + X'Z + YZ$

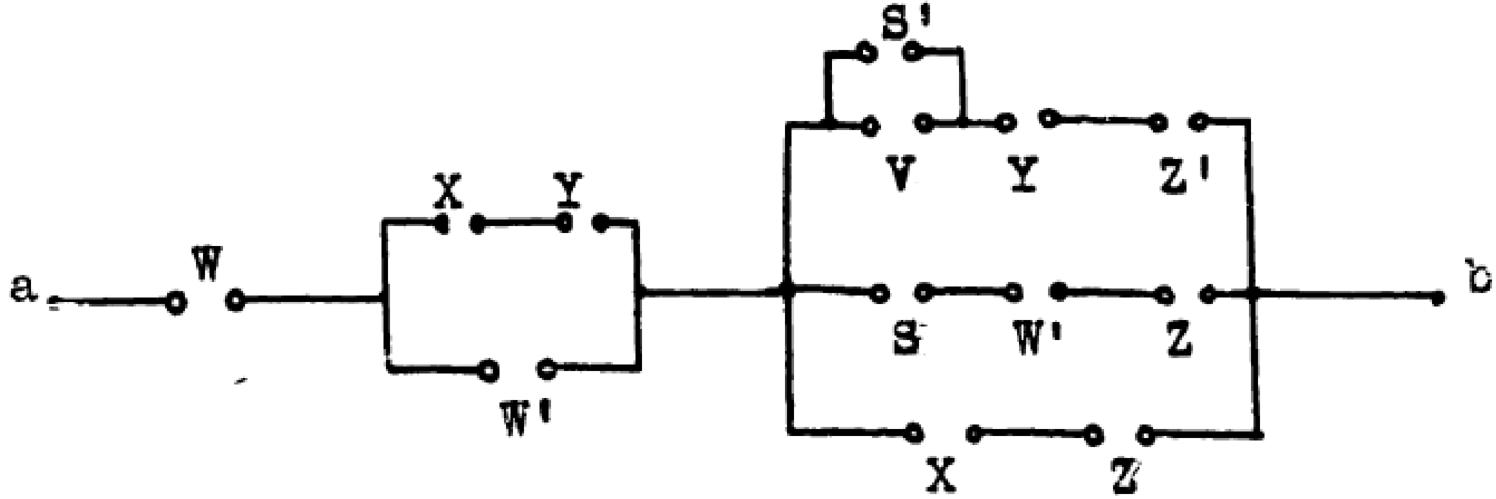
    b. $(X + Y)(X' + Z) = (X + Y)(X' + Z)(Y + Z)$

17. a. $Xf(X) = Xf(1)$

    b. $X + f(X) = X + f(0)$

18. a. $X'f(X) = X'f(0)$

    b. $X' + f(X) = X' + f(1)$



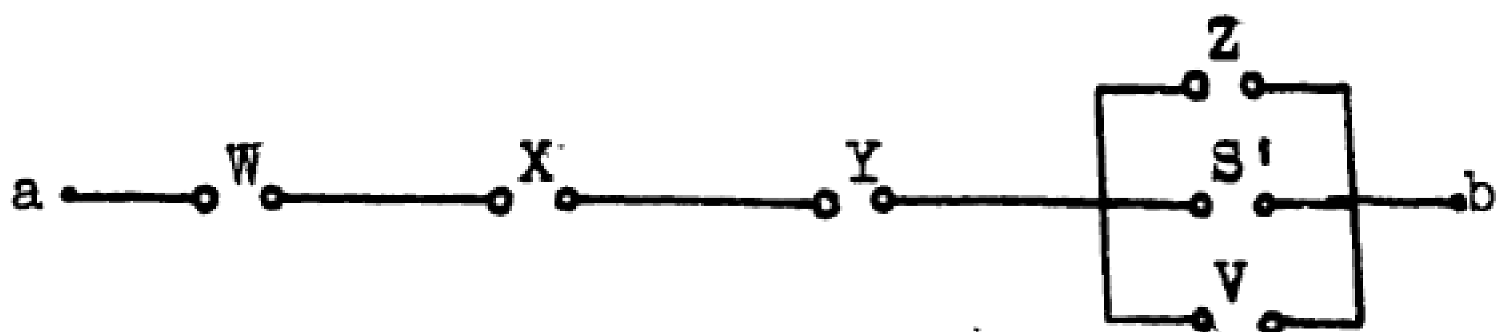
$$X_{ab} = W + W'(X+Y) + (X+Z)(S+W'+Z)(Z'+Y+S'V)$$

$$= W + X + Y + (X+Z)(S+\overset{W'}{1}+Z)(Z'+Y+S'V) \checkmark$$

$$= W + X + Y + Z(Z'+S'V)$$

$$X_{ab} = W + X + Y + ZZ' + ZS'V$$

$$= W + X + Y + ZS'V$$



Other contributions

How to find expressions for relay circuits that are not decomposable in serial and parallel switches, such as the bridge:

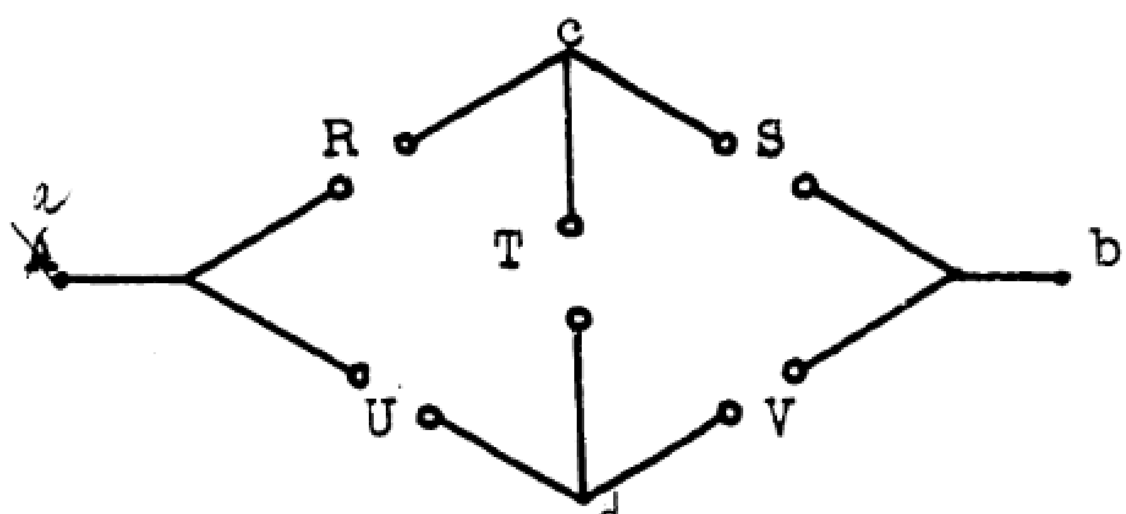


Fig. 11

Other contributions

Analyzing multi-terminal networks by means of matrices:

$$\left\| \begin{matrix} 1 & x'_{12} & x'_{13} & \cdots & x'_{1n} \\ x'_{21} & 1 & x'_{23} & \cdots & x'_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x'_{n1} & x'_{n2} & \cdots & \cdots & 1 \end{matrix} \right\|$$

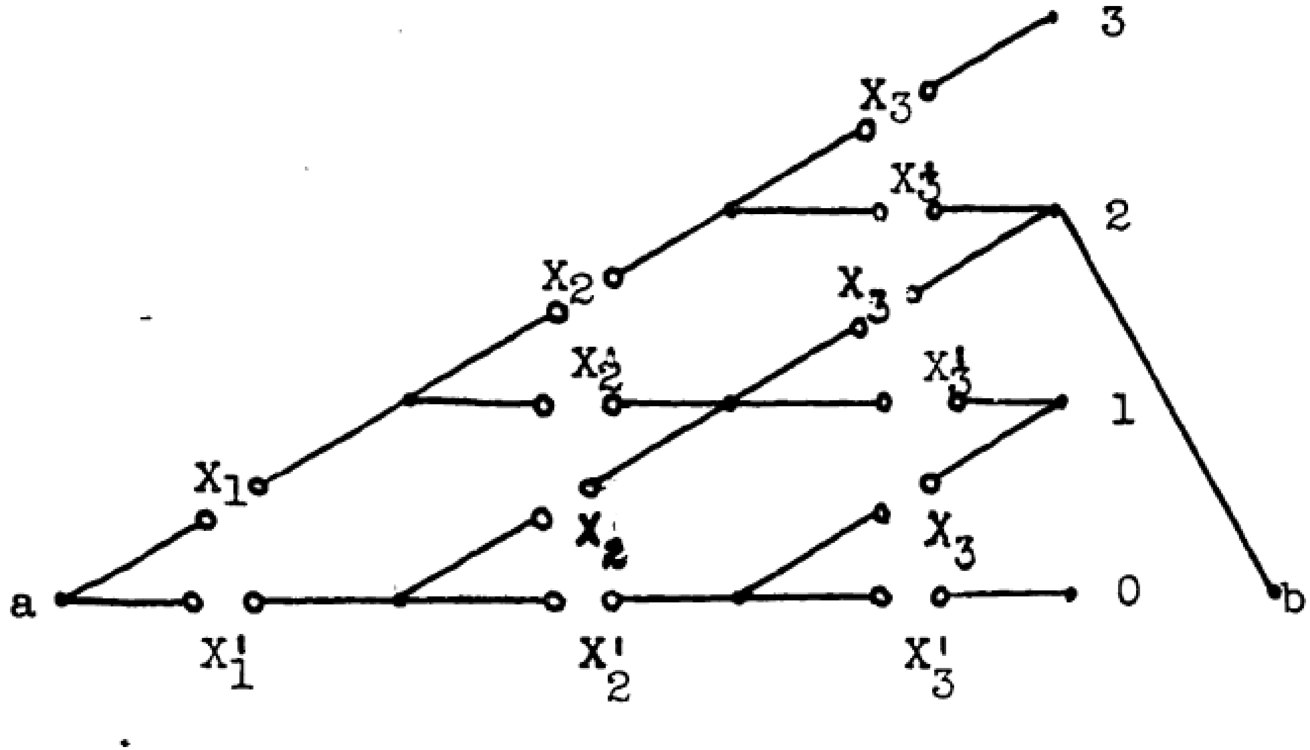
Other contributions

An analysis on the number of functions with N variables, and the complexity of their implementation

Theorem: The number of functions of n variables or less is 2^{2^n}.

Other contributions

An analysis on the complexity of the implementation of symmetric functions



Shannon’s contributions to digital circuit design

- With his MSc thesis, Shannon introduced the use of Boolean algebra in the analysis and design of switching circuits.
- The theoretical rigor of Shannon's work superseded the ad hoc methods in existence.
- His work became the foundation of computer aided digital circuit design:
  - Quine & McCluskey (1952) – Minimum prime cover
  - Espresso (Brayton, 1982) – Efficient two level via transformation into unate functions
  - Espresso-MV (Rudell, 1986) – Generalization for multi-level logic

Shannon’s contributions to computing

- In the process, Shannon described how to build adders, logic gates and circuits that handle conditional decisions.
- Shannon's work pointed the way to electronic computers based on the binary numbering system.
- Together with Alan Turing, Thomas Flowers and John von Neumann, Claude Shannon was one of the founding fathers of digital computing.

Thank you